FORECASTING ABILITY AND THE IMPACTS OF MONETARY POLICY AND EXCHANGE RATE SHOCKS: COMPARISONS BETWEEN DSGE AND VAR MODELS ESTIMATED FOR BRAZIL.

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DOI: 10.47283/244670492020080102

Abstract
This article compares the out-of-sample forecasting ability of a new Keynesian DSGE (Dynamic Stochastic General Equilibrium) model, specified and estimated for Brazil, with a Vector Autoregression (VAR). The article innovates in relation to other similar studies made for Brazil (Castro et al. (2011) e Caetano e Moura (2013), by choosing a specification for the DSGE model that, allowing the use of a richer information set, made possible to compute the predictive ability of the DSGE from forecasts that are, truly, out-of-sample forecasts. Moreover, unlike other articles that used Brazilian data, it also verifies to what degree the responses of variables to a monetary and an exchange rate shock. The estimated DSGE model is similar to the ones adopted by Justiniano e Preston (2005) and Alpanda (2009, 2010a, 2010b). The BVAR model was estimated using a Bayesian procedure like those proposed by Sims and Zha (1998) and Rubio-Ramíres, Waggoner and Zha (2005). The results show that the estimated DSGE model is capable of making out-of-sample forecasts, for some variables, that are competitive when compared to a VAR model.
Keywords: DSGE. Forecast. Impulse response function.

Introduction

In this article, we analyze the predictive ability of an estimated Dynamic Stochastic General Equilibrium Model (DSGE) for Brazil, comparing its predictions with those of a Vector Autoregressive (VAR) model. In order to reduce the probability of structural breaks in the model, due to the frequent changes in economic policy that occurred prior to 1999 (from 1999 onwards, floating exchange rates, inflation targets and primary government deficit targets were adopted), they used data from the first quarter of 1999 to the second quarter of 2013, totaling 58 observations. The models were estimated using Bayesian inference methods - using the Dynare and Matlab programs.

In the specification of the estimated DSGE model for Brazil, the hypothesis of a small and open economy was adopted and the condition of open interest rate parity was modified to allow a negative relation between the country risk premium and the rate expected for domestic currency depreciation (incorporating the forward premium puzzle). The estimated DSGE model is similar to that used by Justiniano and Preston (2005) and Alpanda (2009, 2010a, 2010b). In addition, a Taylor rule was adopted forward looking, whereby the choice of the domestic interest rate by Brazilian Central Bank (BCB) depends on the prediction of future inflation rather than current inflation.

The general stochastic-dynamic equilibrium models, better known in the literature, by DSGE, are part of the frontier of macroeconomic research. In contrast to the traditional

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Keynesian approach, which simply theorizes relations between macroeconomic aggregates, the DSGE models start from micro-foundations in their structure, allowing consistent modeling of expectations and thus not subject to Luke's criticism. Moreover, unlike the Vector Autoregressive (VAR) models, exogenous processes - shocks - have direct economic interpretations, without the need to identify them through the adoption of additional identification hypotheses. The DSGE assumes that the economy is a system in stochastic-dynamic equilibrium that reflects the collective decisions of rational individuals on a set of variables, considering the values of these variables not only in the present but also in the future. The economy is always in equilibrium, and all available information is embodied by agents. Therefore, people make rational choices that are great for them, not making persistent mistakes. The errors that occur are caused by information gap, that is, they are unanticipated shocks to the economy.

The main emphasis of the modern economy, however, is on the individuals' response to shocks and how this affects multiple markets simultaneously, both in the present and in the future. The DSGE models have been effective in describing the dynamics of business cycles and the effects of a wide variety of shocks in the economy (CHRISTIANO et al., 2010). For this reason, the DSGE models are being applied by central banks around the world, in order to aid in policy decisions.

The article innovates in relation to other similar works made for Brazil (Castro et al. 2011) and Caetano and Moura (2013), when choosing a specification for the DSGE model that, by allowing the use of a more information set rich, made it possible to compute the predictive ability of the DSGE from predictions that are truly out-of-sample predictions.

The results obtained here show that the DSGE model is able to generate predictions, for some variables, competitive in relation to other rival models - VAR and Random Walk -, using Theil-U statistics.

1 Literary review

Research on the predictive quality of DSGE models has made recent advances. In the international context, we have established works such as Smets and Wouters (2007 e 2009), which presented a model for the American economy, which incorporates several types of real and nominal frictions, as well as seven types of structural shocks. Still in the context of the US economy, Edge, Kiley and Laforte (2010) compare the forecasts of a detailed specified DSGE model with those of time series models and those of analysts of the Federal Reserve Board. The authors conclude that the DSGE model produces forecasts of macroeconomic aggregates that are competitive with those obtained by more traditional models of central banks, since the out-of-sample forecast of the DSGE model of the Federal Reserve Board exceeds the macroeconometric model and the publication of forecasts of the Greenbook.

Christoffel, Coenen and Warne (2008) investigated the predictive ability of a DSGE model in the euro region. In addition, the authors also evaluated the quality of the model's response to policy changes, concluding that the model's properties are economically plausible, especially in relation to the spread of shocks and the identification of the main sources of economic fluctuations.

With regard to the Brazilian literature, there are few studies related to the predictive analysis of DSGE models. The most complete study is the Castro et al. (2011), prepared by the Central Bank. In this paper, in addition to comparing the quality of the forecasts of several models, the quality of responses of the adverse model types of shocks, including shocks in economic policy, is also analyzed. It is important to note that in such work the estimation of the model is done using the whole sample, since the initial sample contains only 22 observations. In addition, only four observable variables are used in the model predictive ability test: GDP

(Gross Domestic Product) growth rate, inflation rate measured by the IPCA (Consumer Price Index), real exchange rate and the Selic interest rate. The results pointed to a superior performance of the DSGE when compared to the other prediction models. The forecasts of the GDP growth rate and the Selic rate showed high mean squared errors in all models, with a small advantage for the DSGE.

The most recent work is that of Caetano and Moura (2013), which use a hybrid macroeconometric model (DSGE and VAR) for the Brazilian economy. Despite the simplicity of the DSGE model used, the predictions of the hybrid model are generally significantly more accurate than the predictions of first and second order VAR models. In this sense, we conclude that the DSGE model used contributes significantly to improve the predictions of VAR models.

With regard to the estimation of DSGE models and analysis of shocks in the economy, the available national literature is a little richer: Cavalcanti and Vereda (2011) present the dynamic properties of a DSGE model for Brazil, under alternative parameterizations. The authors study the admissible ranges of values of the main parameters of the models and then calculate the impulse-response functions of interest, under various parameters. According to the results obtained, the responses of some of the main macroeconomic variables to the analyzed shocks are compatible with stylized facts for the Brazilian economy and are also reasonably robust to the choice of the structural parameters of the model with regard to its timing, but not in the which relates to its magnitude.

Valli and Carvalho (2010) present a new-Keynesian DSGE model in which the economy is composed of heterogeneous families, which differ in their budgetary restrictions and skills at work. It is a calibrated model, and alternative types of monetary and fiscal policy reaction function are used to analyze the iteration between them. This comparison is made through the impulse response and moment analysis functions.

2 Models

2.1 DSGE

The model deals with a small and open new-Keneysian economy, similar to that presented in the texts of Alpanda et al. (2009, 2010a, 2010b), Steinbach et al. (2009), Justiniano and Preston (2005) and Gali and Monacelli (2005). Families decide on wages in the labor market, while firms, which are monopolistically competitive, decide on prices in the goods market. Nominal rigidities are introduced through price escalation and salary determination to Calvo, as well as the indexation of prices of household goods and wages to past inflation. In addition, firms act in a monopolistic competition scenario, and the production process is formed by firms that produce domestic goods, and also by importing firms, which import goods from abroad and sell such products in the domestic currency market national. In the latter case, as the price determination is made in a step-by-step manner, a partial pass-through of the exchange rate is reached. Other characteristics of the model are: an incomplete distribution of international risk and interest rate overdraft, which is modified as in Adolfson et al. (2008), so that the rate of domestic currency depreciation, both current and expected in the future, changes the country risk premium. Then, the equations that characterize the equilibrium of the model will be described.

2.2 VAR

The classical VAR is represented by a set of equations in which all economic variables are treated as endogenous; thus, the VAR models examine linear relations between each variable and the lagged values of itself and all other variables. It is a model that imposes a small number of constraints: it is only necessary to choose the relevant set of variables and, in the classical version, the maximum number of lags of all variables in all equations. In the classic version, the number of lags is usually chosen based on information criteria such as Akaike or Schwarz.

The VAR model, in reduced form, can be written as follows:

$$y_t = d_t D + \sum_{l=1}^{p} y_{t-1} B_l + u_t \quad \text{(Equation 1)}$$

$$E(u_t | y_1, \ldots, y_{t-1}, d_1, \ldots, d_t) = 0 \quad \text{(Equation 2)}$$

at where y_t is a line vector of endogenous dimension variables$1xn$;B_l are arrays of dimension parameters nxn; D is an arrayqxn of parametersd_t is a vector line $1xq$ of seasonal dummies;p corresponds to the extent of eu_t is a vector waste line.

3 Data and estimation

31 Data

The estimation of the DSGE model makes use of 10 observable variables: product growth, Δy_t; implied deflator variation of GDP, $\pi_{h,t}$; IPCA, π_t; nominal interest rate,i_t; wage inflation (nominal), $\pi_{w,t}$; growth of labor productivity, Δz_t; nominal depreciation of the currency,d_t; foreign product growth, Δy_t^*; variation of the implicit deflator of the GDP foreign, $\Delta \pi_t^*$; and foreign nominal interest rate, Δi_t^*.

The data set is in quarterly frequency and contains 58 observations, including the time period from the first quarter of 1999 to the second quarter of 2013. To calculate out-of-sample forecast errors, the model is recursively estimated using 2008T4 endpoints until 2013T2, with the same starting point, 1999T1. The database will be described below. The series of variation of the GDP had as data source the IBGE (Brazilian Institute of Geography and Statistics). This series is seasonally adjusted and represents the percent change of the quarter in relation to the immediately preceding quarter. The series of variation of the implicit GDP deflator was obtained by dividing the series of current GDP by real GDP, both taken from the IBGE domain, and calculating the variation of the quarter in relation to the immediately previous quarter. It is worth mentioning that the quarterly figures obtained are close to the annual figures provided by IBGE. The series of IPCA is originally of monthly frequency and, to transform it in quarterly, accumulated geometrically the rates of three months; the exchange variation was formed from the monthly nominal commercial exchange rate series, therefore, the quarter rate was considered as the average rate of three months. The data were obtained by IPEA. The nominal interest rate was taken from the website of the Central Bank of Brazil (BCB) and, because it was monthly, the rates were geometrically accumulated to transform it into quarterly. The series used to calculate wage inflation was found on the IBGE website and refers to the variation of the nominal average income of the main work actually received in the reference month by the persons 10 years of age or older employed in the reference week. Since the frequency is monthly, the average of the three months of a quarter was calculated against the average of three months of the previous quarter. The labor productivity growth was calculated by the ratio between the quarterly industrial production series and the employed population in the manufacturing industry in the quarter, with the source of both IBGE. As for the foreign party, the data sources were: Bureau of Economic Analysis (GDP and implicit deflator of US GDP) and Federal Reserve Board (US short-term interest rate).

All series were seasonally adjusted through the Demetra program, and the SARIMA model specification was chosen automatically by the program, using the X-12 SARIMA module. In the following table, the SARIMA models identified by Demetra are presented.

Table 1 – The SARIMA models identified by Demetra.

Series	SARIMA Models

Δy_t	[(0,1,1)(0,1,1)]
$\pi_{h,t}$	[(0,1,1)(0,1,1)]
π_t	[(0,1,1)(0,1,1)]
i_t	[(0,1,1)(0,1,1)]
$\pi_{w,t}$	[(0,1,1)(0,1,1)]
Δz_t	[(0,1,1)(0,1,1)]
d_t	[(0,1,1)(0,1,1)]
Δy_t^*	[(0,1,1)(0,1,1)]
$\Delta \pi_t^*$	[(0,1,1)(0,1,1)]
Δi_t^*	[(0,1,1)(0,1,1)]

Source: The author, 2018.

3.2 Estimation of models

3.2.1 DSGE

The model was estimated using Bayesian techniques, which obtain more satisfactory results than the calibration and are simpler than the maximum likelihood (FERNANDEZ-VILLAVERDE, 2010). Regarding the classical estimation methodology, the probability analysis of an event is given by its relative frequency, and the parameters of the model are considered fixed and unknown. Thus, some methods are used to obtain non-biased sample estimators that converge to their true value. After estimation, the parameters are evaluated by means of hypothesis tests in repeated samples that guarantee that result in a certain probability. On the other hand, in the Bayesian approach, the parameters are considered as random variables, each with its own probability distribution.

This methodology allows the use of available information, in the form of a priori distribution, and the use of the data to update such priori, providing the calculation of the a posteriori distributions of the estimated parameters. Technically, such a procedure helps to model the likelihood function, which is particularly important for the estimation of DSGE models. In practice, the Bayesian method is a bridge between model calibration and classical estimation. The calibration practice does not incorporate the likelihood information, but the likelihood estimation is not technically possible. Bayesian estimation makes it possible to estimate weakly identifiable parameters by incorporating a priori information.

To this end, the recent literature on DSGE models was followed and the model parameters were estimated using Bayesian methods. The dynamic system of linear equations that characterize equilibrium can be summarized by:

$$AE[\xi_{t+1}] + B\xi_t + C\xi_{t-1} + u_t = 0 u_t \sim i.i.d.\ N(0,V) \qquad \text{(Equation 3)}$$

at where ξ_t is the vector of variables; u_t is the vector containing the Gaussian orthogonal shocks; A, B, C are matrices in which the elements are non-linear functions of the structural parameters; and V contains the parameters that define the distribution of shocks in the DSGE model. Given a set of parameter values, Blancharde Kahn's method can be used to find the functions that describe how the variables evolve over time in function of their own past and the current realizations of shocks under the hypothesis of rational expectations:

$$\xi_t = P\xi_{t-1} + Qu_t u_t \sim i.i.d.\ N(0,V) \qquad \text{(Equation 4)}$$

It is worth noting that the coefficients of the matrices P and Q are constrained by the economic structure theoretically imposed to the DSGE model, and that some variables in ξ_t are not observable.

To estimate the DSGE parameters, was treated as the transition equation of the model representation in state space, with the equation of measure of the form $\xi_t^* = M\xi_t$, at where ξ_t^*

is the vector of observable variables, and M is a matrix that encompasses the observable elements of the matrixξ_t. Given the a priori density for the parameters of the DSGE model, $\Upsilon(\phi)$, and the observable series, ξ_t^*, the Bayes rule implies that the posterior distribution of the parameters is proportional to the product of the priori with the likelihood function:

$$\square(\phi|\xi_t^*)\ \alpha\ \square(\phi). L(\xi_t^*|\phi) \qquad \text{(Equation 5)}$$

where the likelihood function, $L(\xi_t^*|\phi)$, can be calculated for any set of parameter values using the Kalman Filter. The fashion of a posteriori distribution can be obtained by using a standard maximization algorithm for the set of all possible parameter values. To construct the complete a posteriori distribution and identify its corresponding moments, the Monte Carlo Markov Chain method must be implemented.

3.2.2 VAR

In a classical VAR, the previous equation is estimated via OLS (ordinary least squares). In this article, using several information criteria, a maximum number of lags equal to two was obtained. In addition, if a larger number of lags were adopted, this would result in a low number of degrees of freedom.

3.3 A posteriori estimation of DSGE model parameters

Before the estimation, three parameters were calibrated: the time discount factor, βit's the same as 0,989, reflecting the inflation target of 4,5%. The parameter of the share of imports in, γ, is 0,13 – it is the average in the analyzed period. The elasticity of substitution between types of differentiated jobs offered by families, ε_w, was set in, consistent with the model chosen with a salary mark-up of 20%.

Establishing a priori for the parameters requires the choice of the type of distribution and the values of its parameters. For the beta distribution, we chose parameters that are in the range [0,1], and for the gamma and the inverse gamma positive parameters were chosen. As the information a priori has a high degree of uncertainty, it was avoided to use tight prioris.

Regarding the choice of parameter priori, the averages of these parameters were chosen to be equal to the calibrated values of the parameters drawn from the works of Justiniano and Preston (2005), Cavalcante and Vereda (2011) and Alpanda (2009, 2010a, 2010b). Authors have done an extensive study of microeconomic data analysis, based on evidence from other countries and knowledge of experts in the subject.

Regarding the estimation of the preference parameters, it is possible to observe that, in relation to a priori of the parameter that measures the risk aversion (or the inverse of the elasticity of intertemporal substitution), σ, the distribution was considered a normal with a mean of 1.5 , consistent with the evidence that this parameter is equal to or greater than 1. The persistence parameter of the consumption habit, ζ, has a beta distribution with 0.7 of the mean, based on the evidence of international work on the response of domestic consumption to shocks monetary policy.

As for the Calvo parameters, an average of 0.5 was assumed for the beta distribution, representing an optimum price adjustment approximately every two quarters. As for the indexing parameters, we also opted for the beta distribution. The value of 0.70 was adopted as an average for the price indexation parameters - which are high due to inertial indexing mechanisms in the country - and also for wages, which are considered rigid by the fact that the minimum wage is readjusted annually in Brazil.

The parameters of the Taylor rule, in turn, have relatively loose priori. The beta distribution was assumed for the parameter of interest rate smoothing with a mean of 0.75 and standard deviation 0.15; for the expected inflation coefficient, we have a gamma distribution centered on 1.5, with a standard deviation of 0.25. For GDP growth, a gamma distribution with a mean of 0.25 and a standard deviation of 0.1 was used. And, with regard to the depreciation coefficient, the same value was adopted as Alpanda (2009, 2010a, 2010b), with gamma distribution of mean 0.12 and standard deviation 0.05.

The persistence parameters of the shocks are represented by a beta distribution with a mean of 0.5 and a standard deviation of 0.2. Finally, the shocks are given from the inverse gamma distribution, with an average of 0.5% and infinite standard deviation.

The Dynare program was used to estimate the model. The posterior distributions for the parameters were calculated using the Metropolis-Hastings algorithm, which is based on a Monte Carlo Markov chain (MCMC) procedure. Table 1 shows the a priori distributions of the parameters and the corresponding estimates for the fashion, mean and the lower and upper limits (10-90%) of the posterior distribution considering the complete sample.

The a posteriori estimates were made based on a relatively high standard deviation and the data are informative for all the parameters, except for the elasticity of the risk premium, χ, which is basically given by the choice of a priori with tight standard deviation.

The value of the posteriori mean of the parameter related to the habit of consumption is close to the estimated mean value with Brazilian data of 0.70. The indexation parameters have the average posteriori estimates well below the a priori average of 0.7, except for the indexation of salaries, which is close to such value. Thus, the model failed to capture the structure of price indexation in Brazil. On the other hand, the Calvo probability parameters - with a priori average of 0.5 - had estimates around this range, and in the case of the domestic price readjustment, the fashion was higher, indicating a frequency of price adjustment to just over three quarters.

The Taylor rule has the very considerable smoothing parameter (with a posteriori fashion equal to 0.70) and places a greater weight on the response to inflation and less on the response of GDP growth, compared to the priorities chosen (with estimates of 1 , 81 and 0.26, respectively). Moreover, the estimation of the depreciation weight in the Taylor rule is 0.05, equal to the a priori average chose.

The inverse elasticity of intertemporal substitution, σ, is estimated at 1.77, which is consistent with the evidence that this parameter is equal to or greater than unity. Both the elasticity of labor supply and the elasticity of substitution between domestic and foreign goods have very inelastic estimates.

Shocks are considered persistent, with the exception of the cost-push shock of foreign goods and wages. In terms of volatility of shocks, a high standard deviation was estimated in the foreign cost-push shock, indicating a large variation of mark-up in the prices of imported goods.

Table 2 - Estimation of parameters

	Structural parameters	Density *a priori*	Mode *a posteriori*	Average *a posteriori*	*Posteriori* (10%, 90%)
ζ	Habit in consumption	B (0,7, 0,15)	0,732	0,712	(0,581, 0,803)
σ	Intertemporal Reverse Elasticity	G (1,5, 0,37)	1,778	1,688	(1,245, 2,323)
ϑ	Reverse elasticity of labor supply	G (2, 0,75)	2,701	2,572	(1,713, 4,233)
η	Elasticity of substitution between goods domestic and foreign	G (1,5, 0,75)	0,521	0,518	(0,499, 0,551)

χ	Elasticity of risk premium	N (0,01, 0,001)	0,009	0,009	(0,008, 0,011)
ϕ	UIP Modification Parameter	B (0,1, 0,2)	0,101	0,104	(0,028, 0,221)
Θ_h	Probability of Calvo: price of goods domestic	B (0,5, 0,15)	0,890	0,890	(0,863, 0,922)
Θ_f	Probability of Calvo: price of goods foreign	B (0,5, 0,15)	0,321	0,332	(0,204, 0,412)
Θ_w	Probability of Calvo: wages	B (0,5, 0,15)	0,546	0,531	(0,473, 0,631)
φ_h	Indexation: price of goods domestic	B (0,7, 0,15)	0,284	0,280	(0,159, 0,465)
φ_f	Indexation: price of goods foreign	B (0,7, 0,15)	0,339	0,275	(0,135, 0,608)
φ_w	Indexation: wages	B (0,7, 0,15)	0,558	0,578	(0,314, 0,792)
Taylor's Rule Parameters					
ρ	Taylor's Rule: Smoothing	B (0,75, 0,15)	0,703	0,694	(0,648, 0,777)
λ_π	Taylor's Rule: Inflation	G (1,5, 0,25)	1,815	1,856	(1,401, 2,148)
λ_y	Taylor's rule: GDP growth	G (0,25, 0,1)	0,269	0,258	(0,136, 0,450)
λ_d	Taylor's Rule: Depreciation	G (0,12, 0,05)	0,053	0,0477	(0,029, 0,081)
Persistence Parameters					
ρ_z	AR(1): productivity	B (0,5, 0,2)	0,667	0,653	(0,545, 0,817)
ρ_c	AR(1): consumption demand	B (0,5, 0,2)	0,945	0,956	(0,899, 0,980)
ρ_h	AR(1): cost-push of domestics goods	B (0,5, 0,2)	0,223	0,205	(0,008, 0,383)
ρ_f	AR(1): cost-push of domestics foreign	B (0,5, 0,2)	0,805	0,811	(0,734, 0,871)
ρ_w	AR(1): cost-push of domestics wages	B (0,5, 0,2)	0,257	0,240	(0,137, 0,436)
ρ_d	AR(1): depreciation (country risk)	B (0,5, 0,2)	0,892	0,896	(0,825, 0,982)
ρ_i	AR(1): monetary policy	B (0,5, 0,2)	0,416	0,396	(0,299, 0,556)
ρ_{y*}	AR(1): foreign production	B (0,5, 0,2)	0,965	0,964	(0,950, 0,989)
$\rho_{\pi*}$	AR(1): foreign inflation	B (0,5, 0,2)	0,563	0,556	(0,408, 0,713)
ρ_{i*}	AR(1): foreign interest rate	B (0,5, 0,2)	0,924	0,920	(0,892, 0,960)
Standard deviation of shocks					
σ_z	Shock i.i.d: productivity	IG (0,5%, ∞)	3,49%	3,39%	(0,30%, 4,02%)

σ_c	Shock i.i.d: consumption demand	IG (0,5%, ∞)	0,16%	0,17%	(0,01%, 0,19%)
σ_h	Shock i.i.d: cost-push of domestics goods	IG (0,5%, ∞)	1,03%	1,01%	(0,09%, 1,21%)
σ_f	Shock i.i.d: cost-push of foreign goods	IG (0,5%, ∞)	22,42%	21,81%	(13,07%, 29,82%)
σ_w	Shock i.i.d: cost-push of wages	IG (0,5%, ∞)	1,75%	1,73%	(0,2%, 2,11%)
σ_d	Shock i.i.d: depreciation	IG (0,5%, ∞)	0,49%	0,05%	(0,31%, 0,72%)
σ_i	Shock i.i.d: monetary policy	IG (0,5%, ∞)	0,40%	0,04%	(0,23%, 0,46%)
σ_{y*}	Shock i.i.d: foreign production	IG (0,5%, ∞)	0,65%	0,66%	(0,55%, 0,73%)
$\sigma_{\pi*}$	Shock i.i.d: foreign inflation	IG (0,5%, ∞)	0,19%	0,19%	(0,17%, 0,22%)
σ_{i*}	Shock i.i.d: foreign interest rate	IG (0,5%, ∞)	0,13%	0,13%	(0,11%, 0,15%)

Source: The author, 2018.

4 Results

To be a good model to assist in the conduct of economic policies, the DSGE should contain relevant information about the future dynamics of Brazil's business cycle. In this way, we will compare forecasts outside the DSGE, VAR and Random Walk models.

The DSGE and VAR models were estimated recursively using endpoints in 2008T4 and until 2013T2, with the same starting point in 1999T1. After each estimation, the models were simulated for eight quarters ahead, starting at the end point of the estimated sample.

The performance of the predictions outside the sample of the models was evaluated through the Theil-U statistic, which is equal to the root division of the root mean squared error of the prediction model by root mean square error of a naive model (random walk without constant term) . Thus, if U = 1 means that the model predicts as well as the naive, if U> 1, the naive model is better and only if U <1 can be said that the estimated model produces more accurate predictions than naive.

$$U = \sqrt{\frac{\sum_{t=1}^{n-1}\left(\frac{F_{t+1}-Y_{t+1}}{Y_t}\right)^2}{\sum_{t=1}^{n-1}\left(\frac{Y_{t+1}-Y_t}{Y_t}\right)^2}} \qquad \text{(Equation 6)}$$

where U is the Theil-U statistic, F is the forecast, and Y is the observed value.

Table 2 shows the Theil-U statistics calculated for the eight quarters ahead of the variables exchange rate, GDP growth, GDP deflator, nominal interest rate, wage inflation and productivity, defined in section 3.1.

Table 3 - Statistics Theil-U

Steps ahead	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**
Exchange rate variation								
DSGE	0,555	0,476	0,404	0,361	0,382	0,426	0,372	0,333
VAR	1,384	1,203	0,803	0,756	0,797	0,838	0,671	0,520

Growth GDP								
DSGE	0,593	0,542	0,430	0,301	0,228	0,233	0,276	0,323
VAR	1,433	0,997	0,898	1,191	0,536	0,568	0,591	0,340
Inflation deflator								
DSGE	1,477	1,545	1,988	1,819	1,245	1,588	1,530	1,177
VAR	2,213	1,667	1,351	1,334	1,014	1,461	1,376	0,692
IPCA								
DSGE	1,618	1,356	1,805	1,623	1,450	1,814	1,662	1,329
VAR	3,858	2,328	2,679	2,159	2,132	2,155	1,618	1,498
Nominal interest								
DSGE	7,485	4,560	3,735	3,497	3,401	3,488	3,729	4,186
VAR	2,353	2,618	2,697	2,315	1,984	1,695	1,486	1,515
Wage inflation								
DSGE	0,784	0,887	0,962	0,850	1,085	0,869	0,900	0,628
VAR	1,166	1,009	1,617	1,435	1,963	1,321	0,885	1,163
Productivity								
DSGE	0,762	0,421	0,359	0,311	0,328	0,362	0,373	0,294
VAR	0,958	0,669	0,891	1,197	0,521	1,422	0,937	0,618

Source: The author, 2018

As can be seen from the previous data, the DSGE model has the best Theil-U statistics. In addition, it also stands out in the forecast of the variables of exchange rate of change and GDP growth. With regard to labor productivity, both models performed well. Thus, it can be concluded that the DSGE may contribute to an improvement in the forecasts of some Brazilian macroeconomic variables.

Conclusion

In this paper, the predictive ability of a New-Keynesian DSGE model specified for Brazil is estimated and analyzed, comparing its predictions with those of a Vector Autoregressive (VAR) model. In order to avoid the existence of structural breaks, due to the frequent changes of policy occurred in Brazil before 1999, all models were used for data from the first quarter of 1999 to the second quarter of 2013.

As for the predictive ability, the DSGE models predict better than the VAR model, according to the Theil-U statistic used. The DSGE stands out in the forecast of the rate of change of the exchange and in the forecast of the rate of growth of the GDP. The BVAR performs better in the forecasts of domestic inflation and interest rates. It is important to note that all predictions of the VAR had a value for the Theil-U statistic less than one, that is, the VAR predicts better than the naive model, as opposed to the DSGE that performed poorly on some variables.

A possible extension of this work would be to investigate which changes in the structure of the DSGE model would increase its predictive ability in the predictions of the nominal variables.

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